Student Name: ____ Pd. ____ Date: __________

Basic Skills Supplement 2

Converting Units with Dimensional Analysis

INFORMATION

Dimensional analysis, also known as the "factor-label" method of unit conversion, is a simplistic visual system for converting from any unit to any other compatible unit (often in ways that you might not expect).

The core concept of dimensional analysis is that units can be converted in a series of steps, with each step consisting of a change from a quantity expressed in one unit to an equivalent quantity expressed in a different unit. Along the way, units of the same kind with "divide out," and the end result will be the unit of interest.

Recall that 1 dozen eggs contains 12 individual eggs, and take the following simple example to determine how many eggs are in 7 dozen:

$$\left(\frac{7\ dozen}{1}\right)\left(\frac{12\ eggs}{1\ dozen}\right) = \text{? eggs}$$

Notice that the starting quantity, 7 dozen, is expressed as a fraction (the denominator is 1 simply because there is nothing else in the denominator). The second term, you should notice, represents an equivalency between the numerator and the denominator – even though the units are different, the quantities are equal.

The quantity "1 dozen" is placed in the denominator because its unit has to "divide out" in order for the final unit to be properly reflected. Note that only the unit divides out – the numerical value (which, in this case, happens to be 1), is left in place:

$$\left(\frac{\cancel{7\ dozen}}{1}\right)\left(\frac{12\ eggs}{\cancel{1\ dozen}}\right) = \text{? eggs}$$

See now that "dozens" have been removed from our setup, leaving us only with "eggs" in the numerator (which is the desired unit for this conversion).

The math of dimensional analysis is now similar to any math you may have done in your algebra classes – when multiplying fractions, as in this case, simply determine the product of all of the numerators, determine the product of all of the denominators, and then divide the numerator product by the denominator product to get a final answer.

Thus:

$$\left(\frac{\cancel{7\ dozen}}{1}\right)\left(\frac{12\ eggs}{\cancel{1\ dozen}}\right) = \left(\frac{84\ eggs}{1}\right) = \textbf{84 eggs}$$

It is vitally important that all of the units divide out *except* for the unit you are interested in. To do this, you must be sure that the *unit in the numerator of the current term ends up in the denominator of the following term* in order for them to cancel out. Notice that "dozen" appears in the numerator of the first term but in the denominator of the second term.

This technique can be used in any number of steps, not just in two as in the example above. Look at the following example:

2.5 days = ? seconds

To use dimensional analysis, we start by setting up the given value as a fraction:

$$\left(\frac{2.5\ days}{1}\right) = ?\ \text{seconds}$$

And then use whatever conversion factors we have available to make the conversion and divide out the appropriate units:

$$\left(\frac{\cancel{2.5\ days}}{1}\right)\left(\frac{\cancel{24\ hrs}}{\cancel{1\ day}}\right)\left(\frac{\cancel{60\ \text{min}}}{\cancel{1\ hr}}\right)\left(\frac{60\ \text{sec}}{\cancel{1\ \text{min}}}\right) = ?\ \text{seconds}$$

Then solve:

$$\left(\frac{\cancel{2.5\ days}}{1}\right)\left(\frac{\cancel{24\ hrs}}{\cancel{1\ day}}\right)\left(\frac{\cancel{60\ \text{min}}}{\cancel{1\ hr}}\right)\left(\frac{60\ \text{sec}}{\cancel{1\ \text{min}}}\right) = \left(\frac{216000\ \text{sec}}{1}\right) = 216000\ \text{seconds} = 2.16 \times 10^5\ \text{seconds}$$

Dimension Analysis with Derived Units

Imagine you are working in the laboratory one day and you need to know the density of some substance in kilograms per cubic meter, but your reference material only provides density (for some strange reason) information in cg/mL. Have no fear – dimensional analysis can get the job done!

Let's assume that the density in the reference book is 435.6 cg/mL, and you want to determine this value in kg/m^3. As above, start with the given in the form of a fraction:

$$\left(\frac{435.6\ mg}{1\ mL}\right) = ?\ \text{kg/m}^3$$

Notice that the "mL" value is in the denominator, with a 1 attached to it. This is because mL is in the denominator in a density unit, and the assumption is that this density is 435.6 cg per *1 mL*.

Both the numerator and the denominator can be converted using dimensional analysis. Let's start with the familiar numerator:

$$\left(\frac{\cancel{435.6\,cg}}{1\,mL}\right)\left(\frac{\cancel{1}\,g}{\cancel{100\,cg}}\right)\left(\frac{1\,kg}{\cancel{1000\,g}}\right) = ?\ \text{kg/m}^3$$

So far, we've arrived at kilograms in the numerator. Dealing with the mL unit in the denominator is exactly the same in terms of technique – the only difference is that in this case, the *unit in the denominator of the current term must become the unit in the numerator of the following term, in order for them to divide out*.

Recall that 1 mL = 1 cm^3, and that there are one million (100^3) cubic centimeters in a cubic meter. Observe:

$$\left(\frac{\cancel{435.6\,mg}}{\cancel{1\,mL}}\right)\left(\frac{\cancel{1\,g}}{\cancel{100\,cg}}\right)\left(\frac{1\,kg}{\cancel{1000\,g}}\right)\left(\frac{\cancel{1\,mL}}{\cancel{1\,cm^3}}\right)\left(\frac{\cancel{10^6\,cm^3}}{1\,m^3}\right) = ?\ \text{kg/m}^3$$

Now, we have kilograms in the numerator, and cubic meters in the denominator. Solve:

$$\left(\frac{\cancel{435.6\,mg}}{\cancel{1\,mL}}\right)\left(\frac{\cancel{1\,g}}{\cancel{100\,cg}}\right)\left(\frac{1\,kg}{\cancel{1000\,g}}\right)\left(\frac{\cancel{1\,mL}}{\cancel{1\,cm^3}}\right)\left(\frac{\cancel{10^6\,cm^3}}{1\,m^3}\right) = \left(\frac{435600000\,kg}{100000\,m^3}\right) = 4356\ \text{kg/m}^3$$

This same technique will work with any unit conversion, with any units, as long as the protocols set forth here are strictly followed.

To summarize:

1. Begin by setting up the given value as a fraction.
2. Add a second term, being sure that the unit in the numerator of the first term ends up in the denominator of the second term, so that it can divide out (or vice versa, if the starting unit is in the denominator).
3. Be sure that the *quantities* in the numerator and denominator of each term are equivalent, even though their units are different.
4. Be sure to stop adding terms when you reach the unit of interest!
5. Cancel out any units that can be.
6. Solve.

Exercises

Use the following information for questions 1 through 4:

3 blarps = 7 glorps 8 glorps = 17 trangs 11 trangs = 5 slurps 2 slurps = 23 kilps

1. How many slurps are in 107 kilps?
2. How many blarps are in 73 trangs?
3. How many kilps are in 3 blarps?
4. How many glorps are in 245 trangs?

For the following questions, consult any reliable reference you have available for unit equivalencies.
Convert:

5. 23.5 kilometers to miles.
6. 34.6 kilograms to micrograms.
7. 22.1 m/s to mi/hr.
8. 2.2 light years into furlongs.
9. 3.5 days to seconds
10. 12.2 km to centimeters
11. 2321 minutes, 10 seconds to years
12. 22 inches to yards
13. 45 km/day to inches/week
14. 12 mi/hr to inches/millisecond